The complexity of detecting Rainbow Ortho-Convex 4-Sets in Colored Point Sets

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The *Rectilinear Convex Hull* of a point set P on the plane can be defined in terms of a set of empty open quadrants. An *open quadrant* is the intersection of two open half-planes whose supporting lines are parallel to the x and y axes, and its *apex* p is the intersection point of these supporting lines. Given a point set P, an open quadrant is said to be *empty* if it contains no elements of P. The Rectilinear Convex Hull of P is then the region obtained by removing from $\mathbb{R}^2$ all the empty open quadrants.

Let P be a k-colored set of n points in the plane, $4 \leq k \leq n$. We study the problem of deciding whether P contains a subset of four points of different colors such that its Rectilinear Convex Hull has positive area. We show this problem to be equivalent to deciding if there exists a point c in the plane such that each of the open quadrants defined by c contains a point of P, each of them having a different color.

We provide an $O(n \log n)$-time algorithm for this problem, where the hidden constant does not depend on k; then, we prove that this problem has time complexity $\Omega(n \log n)$ in the algebraic computation tree model. General position assumptions for P are not required.

This talk, refering to [7], will cover the key geometric insights behind the problem equivalence, a description of the algorithmic approach (based on the *line sweeping* paradigm), and the ideas on the lower bound construction (which use Ben-Or's Lower Bound Theorem [5], as well as linear time reductions). We will also present a new variant on the problem and discuss recent progress in its solution.

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