Abstract

Parabolic Trough solar fields are among the most prominent methods for harnessing solar energy. However, continuous sun-tracking movements leads to wear and degradation of the tracking system, raising the question of whether the rotations can be minimized without compromising energy capture. In this paper, we address this question by exploring two problems: (1) minimizing the number of SCA rotational movements while maintaining energy production within a specified range, and (2) maximizing energy capture when the number of rotations is limited. Unlike prior work, we develop a general framework that considers variable conditions. By transforming the problem into grid-based path optimization, we design polynomial-time algorithms that can operate independently of the weather throughout the day.

Optimization of solar tracking 3D-irradiance functions via shortest paths in rectangular grids: an efficient algorithm for energy harvesting

José-Miguel Díaz-Bañez*[1], José Manuel Higes López †[1], Miguel Angel Pérez Cutiño.‡[2], and Tom Todtenhaupt §[2]

[1]Departmento de Matemática Aplicada II, Universidad de Sevilla, España
[2]Virtualmechanics S.L, Sevilla, ESPAÑA

1 Introduction

Parabolic trough collector (PTC) systems are one of the most widespread types of Concentrated solar power (CSP) plants. PTC are built by a cylindrical heat collector element (HCE) and a parabolic surface made with mirrors that concentrates the rays of the sun in the HCE. The HCE carries a heat transfer fluid that is used to produce the steam that feeds a power block to produce electricity. The parabolic surface and the HCE form a Solar collector element (SCE) (see Figure 1) and four SCE are a solar collector assembly (SCA).

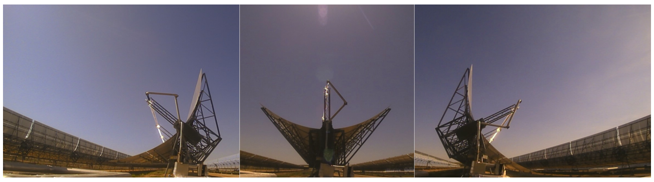

Figure 1: Example of solar tracking in a real PTC.

SCAs are automated to follow the sun in order to collect the maximum energy. Moving SCAs produces mechanical stress on the structure increasing the risk of breaking its components. Hence, reducing the movements of the SCAs while maintaining high levels of energy production is of vital importance for the industry. There have been another approaches to face similar problems in other types of solar plants as for example[1] or [2]. But as far as we know there has not been any attempt to attack this problem in SCAs.

In [3] the authors proposed efficient algorithms to minimize the number of rotational movements of the SCA while maintaining the production within a given range, or to capture the highest amount of energy for a limited number of movements. In that work, it was assumed that the solar irradiance function over the HCE remained constant throughout the day, unaffected by weather conditions. Consequently, the irradiance function was modeled as a 2D function. In this study, we aim to remove this restriction by considering a 3D solar irradiance function, providing a more realistic representation for industrial applications. We will prove that the optimization problems are equivalent to solving specific problems in grid graphs.

2 Problem Definition

Let $f : X \times Y \to \mathbb{R}$ be a function representing the solar irradiance. $x \in X$ is the angular position of the SCA with respect to the plane, and $y \in Y$ the angular position of the sun. We will assume that $Y = [0, 180)$ and $X = [0, 180)$.

Note that, since the angular position of the sun varies at a constant rate, the variable Y can also be interpreted as time. Accordingly, in what follows, we will denote the coordinates in Y interchangeably by t or y. Therefore a movement pattern of the SCA can be represented as a curve in $X \times Y$ that is continuous (as a curve) and it is non decreasing in the y-coordinate. In practice, we may assume that the SCA moves instantaneously and remains stationary for a period of time to collect energy after each movement, i.e. the x-coordinate remains constant while the y-coordinate varies. This motivates the following definition:

Definition 1 *Given a solar irradiance function f, we will say that a sequence of points $\mathcal{P} = \{P_i = (p_i, t_i)\}_{i=1}^{m} \subset X \times Y$ is a generalized collector path (GCP) if $P_1 = (0, 0)$ and $t_m = 180$ and for every $1 < i < m$: (1) $P_i \neq P_{i-1}$ and $P_i \neq P_{i+1}$, (2) $p_i = p_{i+1}$ or $p_i = p_{i-1}$ but not both , (3) $t_i = t_{i+1}$ or $t_i = t_{i-1}$ but not both and (4) $t_i \geq t_{i-1}$. An increasing collector path (ICP) would be defined as a GCP that also satisfies $p_i \leq p_{i+1}$ for every $1 \leq i < m$.*

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†Email: jhiges@us.es. Research partially supported by grants PID2020-114154RB-I00 and TED2021-129182B-I00 funded by MCIN/AEI/10.13039/501100011033 and the European Union NextGenerationEU/PRTR.

‡Email: m.perez@virtualmech.com. Research partially supported by grants PID2020-114154RB-I00 and TED2021-129182B-I00 funded by MCIN/AEI/10.13039/501100011033 and the European Union NextGenerationEU/PRTR.

§Email: t.todtenhaupt@hotmail.de.

A GCP can be visualized as a snake-type movement such that no decreasing movements in the y coordinate are allowed (see Figure 2 left). In an ICP no left movements would be allowed. If $p_i = p_{i+1}$, then the collector is static collecting energy during the period of time $[t_i, t_{i+1})$. If $t_i = t_{i+1}$, then the collector has been moved (instantaneously) $p_{i+1} - p_i$ degrees.

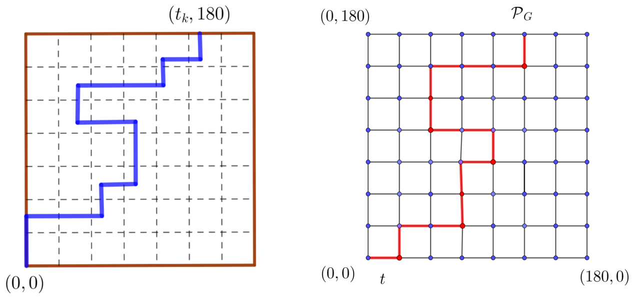

Figure 2: Left: in blue an example of a GCP. Right: in red a $\mathcal{P}_G$ with nine turn points. Red points are vertical points.

The set of points of $X \times Y$ in which the collector is static will be denoted by $\mathcal{S}_\mathcal{P}$. Under these definitions, *the energy* $E(\mathcal{P})$ collected by a GCP, $\mathcal{P}$, is defined as:

$$E(\mathcal{P}) = \sum_{i=1}^{m} \int_{t_i}^{t_{i+1}} f(p_i, y)dy$$

Given a GCP $\mathcal{P}$, we could define the number $m_\mathcal{P}$ of movements of $\mathcal{P}$ as the number of times such that $p_i \neq p_{i+1}$.

Problem 1 *(3D Minimum Tracking Motion) Given an irradiance function f, and two positive real numbers $u_1 < u_2$, find a GCP $\mathcal{P}$ such that $u_1 \leq f(P) \leq u_2$ for every $P \in \mathcal{S}_\mathcal{P}$, and the number of movements of $\mathcal{P}$ is minimum.*

Problem 2 *(3D Maximum Energy Collection) Given an irradiance function f, and a positive integer number m, find a GCP $\mathcal{P}$ with number of movements at most m such that $E(\mathcal{P})$ is maximum.*

3 Equivalence to grid graph problems

As rays are concentrated to a circle and not to a point, there is a short period of time t for which: 1) for a fixed position of the SCA the sun moves without affecting the concentration of rays in the absorber tube; 2) for a fixed position of the sun the SCA moves obtaining the same effect described before. Hence $X \times Y$ can be decomposed into cells $[x_i, x_{i+1}) \times [y_j, y_{j+1})$ of size $t \times t$ where the irradiance function f is constant for each of these cells. Therefore, we can consider *the weighted grid graph* $G = \langle V, E, W \rangle$ as the graph such that $V = \{(x_i, y_j)|$ for every $1 \leq i, j \leq n\}$, E is the set of linear segments that connect two adjacent points of V in the grid, i.e. E is the set of linear segments needed to get a rectangular grid graph with V as the vertex set. And W is the function $W : V \to \mathbb{R}$ defined as $W((x_i, y_j)) = f(x_i, y_j)$.

Given a weighted grid graph G associated to the SCA, we define a restricted generalized collector path $\mathcal{P}$ (R-GCP) as a GCP such that every (p_i, t_i) of $\mathcal{P}$ is a vertex of G. Analogously we can define an R-ICP. Given $\mathcal{P}$ a R-GCP, it can be seen as a path $\mathcal{P}_G$ in the graph G by adding all the vertices of G that are necessary (geodesically) to connect two consecutive points of $\mathcal{P}$. *A turn* in $\mathcal{P}_G$ is a set of 3 consecutive non-colinear vertices. The number of turn points will be denoted as $\Lambda_\mathcal{P}$. We will say that a point is *vertical* if the following vertex to it has greater y coordinate (See Figure 2 right). The set of all vertical points of $\mathcal{P}_G$ will be noted as $\perp_\mathcal{P}$.

Problem 3 *(3D Minimum Tracking Motion for grids, or 3D-MTM): Given two real numbers u_1, u_2 and a weigthed grid graph G, find a R-GCP $\mathcal{P}$ such that $u_1 \leq w_P \leq u_2$, for every $P \in \perp_\mathcal{P}$ and $\Lambda_\mathcal{P}$ is of minimum cardinality.*

Problem 4 *(3D Maximal Energy Collection for grids, or 3D-MEC): Given an integer m and a graph G, find a R-GCP path $\mathcal{P}$ such that $\Lambda_\mathcal{P} = m$ and $\sum_{P \in \perp_\mathcal{P}} w_P$ is maximal.*

One of the main results of our work.

Theorem 2 *Solving problems 1 and 2 is equivalent to solving problems 3 and 4, respectively.*

4 Algorithms

Using dynamic programming, problems 3D-MTM and problem 3D-MEC can be solved in polynomial time.

Theorem 3 *3D-MTM problem can be solved in $O(n^2)$ time and 3D-MEC problem in $O(n^2 m)$ time.*

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